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Modeling Distribution of Jobs Using Production Function

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ABSTRACT

This paper presents a theory in which the composition of jobs is always sub-optimal, we use a search framework which is a natural tool to model a situation in which identical workers can end up in different jobs with very different compensation patterns. First we will show that if different types of jobs have different creation (capital) costs, then those which cost more to create will have to pay higher wages due to rent sharing; therefore. There will naturally exist good and bad jobs in this economy. Second, establish that in an unregulated market, the composition of jobs is in efficiently biased towards bad jobs.

Keywords: production function, equilibrium, compensation patterns, capital equipment.

1. Introduction

One of the most striking and robust stylized facts of labor markets is the presence of persistent and large wage differentials among identical workers in different industries and occupations.

This paper offers a model of the interaction between composition of jobs and labor market regulation. Ex-post rent-sharing due to search frictions implies that 'good' jobs which have higher creation costs must pay higher wages. This wage differential distorts the composition of jobs, and in the unregulated equilibrium there are too many bad jobs relative to the number of good jobs. Minimum wages and unemployment insurance encourage workers to wait for higher wages, and therefore induce firms to shift the composition of employment towards good jobs. As a result, such regulations, even though they will often increase unemployment, will increase average labor productivity and may improve welfare.

This paper offers a theory which explains why good and bad jobs exist. The main ingredient of the theory is the plausible assumption that workers have to search for jobs, and wages are determined by some rent-sharing arrangement. Different industries and occupations use different capital equipment. Those jobs which require more expensive investments, i.e. those which cost more to create, will have to generate more rents ex post to cover their expenses. Rent sharing then implies that workers employed in these jobs will be able to obtain higher wages. In this economy good jobs will thus be those which cost more to create. Workers will search for jobs aware of the presence of wage dispersion, but will accept bad as well as good jobs if the wage differentials not too large.

An immediate implication of this theory concerns the efficiency of job composition, because firms opening good jobs do not take into account the higher rents (thus higher utility), they provide their workers with, the composition of jobs will be inefficiently biased towards bad jobs.

2. Theory

There are three produced commodities. Labor and capital are used to produce two non-storable intermediate goods which are then sold in a competitive market and immediately transformed into the final consumption good of this economy. Preferences of all agents are defined over the final consumption good alone. Throughout the paper, we will normalize the price of the final good to 1.

There is a continuum of identical workers with measure normalized to 1. All workers are infinitely lived and risk neutral. The assumption that workers are risk-neutral obviously leaves out the most important role of unemployment insurance, but it also helps to highlight that the impact of unemployment benefits on job composition is distinct from their insurance role. They derive utility from the consumption of the unique final good and maximize the present discounted value of their utility. Time is continuous and the discount rate of workers is equal to *r*. On the other side of the market, there is a larger continuum of firms which are also risk-neutral with discount rate *r*.

The technology of production for the final good is:

$$Y = AY_g^{\alpha} Y_b^{1-\alpha} \tag{2.1}$$

where Y_g is the aggregate production of the first input, and Y_b is the aggregate production of the second input. The reason for the use of the subscripts *g* and *b* will become clear later. This formulation captures the idea that, there is some need for diversity in overall consumption/production. It is also equivalent to assuming that (2.1) is the utility function defined over the two goods.

Since good and bad inputs are sold in competitive markets, their prices are:

$$p_g = \alpha A \left[\frac{Y_b}{Y_g}\right]^{1-\alpha} \text{ and } p_b = (1-\alpha) A \left[\frac{Y_g}{Y_b}\right]^{\alpha}$$

The technology of production for the inputs is Leontieff. One worker and one firm with an equipment (capital) that costs k_g will produce 1 unit of the first input, and one worker with a firm with the equipment that costs k_b will produce 1 unit of the second input. Since utility is linear whether we think of k_b and k_g as capital costs or not is immaterial. Throughout the paper, we assume that $k_g > k_b$.

Before we move to the search economy, it is useful to consider the perfectly competitive economy. In this case, all workers will sell their labor services at some wage *w*. Since capital costs are higher in the production of one of the inputs, that is $k_g > k_b$, in equilibrium, we will have $p_g > p_b$. But firms irrespective of their sector will hire workers at the same wage, *w*. Thus, there will be neither wage differences nor bad nor good jobs.

Forms and workers come together via a matching technology *M(u,v)* where *u* is the unemployment rate, and v is the vacancy rate (the number of vacancies). The underlying assumption here is that both types of vacancies have the same probability of meeting workers, thus it is the total number of vacancies that enters the matching function. *M(u, v)* is twice differentiable and increasing in its arguments and exhibits constants returns to scale. The flow rate of match for vacancy as $\frac{M(u,v)}{v} = q(\theta)$ where *q(.)* is a differentiable decreasing function and $\theta = \frac{v}{u}$ is the tightness of the labor market. It is immediately follows from the constant returns to scale assumption that the flow rate of match for an unemployed worker is $\frac{M(u,v)}{v} = \theta q(\theta)$. We make the standard assumptions on *M(u, v)* which ensure that $\theta q(\theta)$ is increasing in θ, and that $\lim_{\theta \to 0} q(\theta)\theta = \infty$.

Finally we assume that all jobs come to an end at the exogenous rate *s*, and that there is free entry into both good and bad job vacancies, therefore both types of vacancies should expect zero profit.

Next, we denote the flow return from unemployment by *z* which will be thought as the level of unemployment benefit. We assume that wages are determined by bargaining a la Rubenstein-shaked-Sutton whereby $w = \min\{y - \overline{\pi}, \max(\beta y, \overline{w})\}$ where *y* is the value of output produced, $\overline{w}$ is the outside option of the worker and $\overline{\pi}$ is the outside option of the firm. In other words, wages are equal to a constant proportion of output unless the outside option of one of the parties binds. When a party would do better by not taking part in the employment relation, i.e. when his (its) outside option binds, he (it) receives the outside option, and the other party gets the rest of output. The important feature is the presence of some bargaining in wage determination so that the rents created by jobs are shared with the workers.

Firms can choose either one of two types of vacancies: (i) a vacancy for a intermediate good 1 – a good job; (ii) a vacancy for an intermediate good 2 – a bad job. Therefore, before opening the vacancy a firm has to decide which input it will produce, and at this point, it will have to incur the creation cost, k_b or k_g. Throughout this section we assume that these costs have to be incurred when the firm opens the vacancy and are not recovered thereafter. This is a reasonable assumption since k corresponds in reality to the costs of machinery (which are sector and occupation specific), and investments in know how. Also, as commented above, both types of vacancies face the same probability of meeting a worker, and produce a flow of 1 unit of their respective goods if filled. Therefore, to reiterate the main point: the only difference between these two jobs is that the first good has higher creation costs than the second.

We will solve the model via a series of Bellman equations. We denote the discounted value of a vacancy by J^V, of a filled job by J^F, of being unemployed by J^U and of being employed by J^E. We will use subscripts g and b to denote good and bad jobs. We also denote the proportion of bad job vacancies among all vacancies by ϕ. Then:

$$rJ^U - \dot{J}^U = z + \theta q(\theta)\left[\phi J_b^E + (1-\phi)J_g^E - J^U\right] \tag{2.2}$$

Being unemployed is similar to holding an asset; this asset pays a dividend of z, the unemployment benefit, and has a probability $\theta q(\theta)\phi$ of being transformed into a bad job in which case, the worker obtains J_b^E, the asset value of being employed in a bad job, and loses J^U; it also has a probability $\theta q(\theta)(1-\phi)$ of being transformed into a good job. Finally, during the short instant that the worker is holding this asset, it can appreciate or depreciate in value (basically because some of these variables like θ or ϕ will be different in the future), and hence the term $\dot{J}^U$.

Note that this equation is written under the implicit assumption that workers will not turn down jobs. If p_b were sufficiently small relative to p_g, workers would not take bad jobs. However, in this case, from the optimization of firms, there would be no bad jobs, i.e. $Y_b = 0$, and the price of their output, p_b, will be infinite. Thus this additional qualification is ignored. There is another possibility: if $rJ^U = \beta p_b$, then workers may accept these jobs with some probability $\zeta < 1$. However, such an allocation cannot be a stable equilibrium: if a small measure of bad jobs close their vacancies, this would imply a lower Y_b, and thus p_b would increase and all workers would accept these jobs with probability 1. Since, we are only interested in stable equilibria, we ignore this possibility.

Next, the firm's outside option will never be binding because it is equal to zero from the free-entry condition $\left(J_i^V = 0\right)$. Also, the worker's outside option is simply rJ^U. The equilibrium wages are determined as:

$$w_i = \max\left\{\beta p_i, rJ^U\right\} \tag{2.3}$$

In other words, the worker gets a proportion β of the return of the job, p_i, unless βp_i happens to be less than his outside option rJ^U. Then, the discounted present value of employment can be written as:

$$rJ_i^E - \dot{J}_i^E = \max\left\{rJ^U, \beta p_i\right\} + s\left(J^U - J_i^E\right) \tag{2.4}$$

for $i = b, g$. (2.4) has a similar intuition to J^U,

Since, when matched, both vacancies produce 1 unit of their goods, we also have for $i = b, g$.:

$$rJ_i^F - \dot{J}_i^F = p_i - \max\left\{rJ^U, \beta p_i\right\} + s\left(J_i^V - J_i^F\right) \tag{2.5}$$

$$rJ_i^V - \dot{J}_i^V = -\gamma_i + q(\theta)\left[J_i^F - J_i^V\right] = 0 \tag{2.6}$$

where the last equality that $J_i^V = 0$ is from the free-entry condition.

Because both types of vacancies meet workers at the same rate, and in equilibrium a worker will accept both types of jobs, we have $\frac{Y_b}{Y_g} = \frac{\phi}{1-\phi}$. Thus, we can determine the product prices (and the value of production) of the two inputs as

$$p_g = \alpha A\left[\frac{\phi}{1-\phi}\right]^{1-\alpha} \quad \text{and} \quad p_b = (1-\alpha)A\left[\frac{1-\phi}{\phi}\right]^{\alpha} \tag{2.7}$$

The evolution of the unemployment rate is given by:

$$\dot{u} = s(1-u) - \theta q(\theta) u \tag{2.8}$$

In words, the change in unemployment is equal to the flow of workers into unemployment (due to destroyed jobs) minus the creation of new employment relations.

A steady state equilibrium is defined as a proportion ϕ of bad jobs, tightness of the labor market θ, outside option of workers J^U, prices for the two goods, p_b and p_g such that equations (2.2), (2.4), (2.5), (2,6) and (2.7) are satisfied with $\dot{J}^U = \dot{J}_i^E = \dot{J}_i^F = 0$. Then wages are given by (2.3) and the unemployment rate by (2.8) with $\dot{u}$ set equal to 0.

Proposition 1. A steady state equilibrium always exists and is characterized by (2.2), (2.4), (2.5), (2.6) and (2.7). In equilibrium, for all $k_g > k_b$, we have $p_g > p_b$ and $w_g > w_b$.

Prove:

Let first multiply the equation for J_b^E by ϕ and the equation for J_g^E by $(1-\phi)$, then add these two together and subtract (2.2). This gives:

$$\phi J_b^E + (1-\phi)J_g^E - J^U = \frac{\phi w_b + (1-\phi)w_g - z}{r + s + q(\theta)\theta}$$

which then implies:

$$rJ^U = \frac{(r+s)z + q(\theta)\theta\left[\phi \max\left\{rJ^U, \beta p_b\right\} + (1-\phi)\max\left\{rJ^U, \beta p_g\right\}\right]}{r + s + q(\theta)\theta} \tag{2.9}$$

rJ^U can now be expressed as a function of the two endogenous variables in (2.9):

$$rJ^U = G(\theta, \phi) \tag{2.10}$$

Where G(., .) is continuous in both of its arguments. It can easily be verified that G(., .) is weakly decreasing in ϕ and strictly increasing in θ. Intuitively, as the tightness of the labor market, θ, increases workers find jobs faster, thus rJ^U is higher. Also as ϕ decreases, the proportion of good jobs among the open vacancies increases, and since $w_g > w_b$, the value of being unemployed increases. The dependence of rJ^U on ϕ is the general equilibrium effect mentioned in the introduction: as the composition of jobs changes, the option value of being unemployed changes too.

Next, by combining (2.5) and (2.6), we have:

$$\frac{k_i}{q(\theta)} = \frac{p_i - \max\{rJ^U, \beta p_i\}}{r+s} \tag{2.11}$$

for $i = b, g$.

It is straightforward to see from (2.11) that because $k_g > k_b$, as the competitive equilibrium, we have $p_g > p_b$; the output of good jobs will sell for a higher price. More importantly, this difference in prices immediately implies from the wage determination rule, (2.3), that $w_g > w_b$, thus good jobs will pay higher wages than bad jobs. Good jobs are the ones which cost more to create (or have higher capital costs). Because of this higher cost, they sell for a higher price, and this, due to rent-sharing, leads to higher wages. Now using (2.10), and rearranging:

$$\left(\alpha A\left[\frac{\phi}{1-\phi}\right]^{1-\alpha} - \max\left\{G(\theta,\phi), \beta\alpha A\left[\frac{\phi}{1-\phi}\right]^{1-\alpha}\right\}\right) = \frac{(r+s)k_g}{q(\theta)} \tag{2.12}$$

and

$$\left((1-\alpha)A\left[\frac{1-\phi}{\phi}\right]^{\alpha} - \max\left\{G(\theta,\phi), \beta(1-\alpha)A\left[\frac{1-\phi}{\phi}\right]^{\alpha}\right\}\right) = \frac{(r+s)k_b}{q(\theta)} \tag{2.13}$$

Consider these two equations (2.12) and (2.13) in the $\theta - \phi$ plane. It is easy to check that (2.12), the good job equilibrium locus, along which a firm that opens a good job vacancy makes zero-profits, is upward sloping (see Figure 2.1): a higher value of ϕ increases the left hand side, thus θ needs to change to increase the right hand side (and reduce the left hand side through $G(\theta,\phi)$). Intuitively, a higher value of ϕ implies that p_g goes up from (2.7), and this makes the creation of good jobs more profitable, and thus θ needs to increase to equilibrate the market.

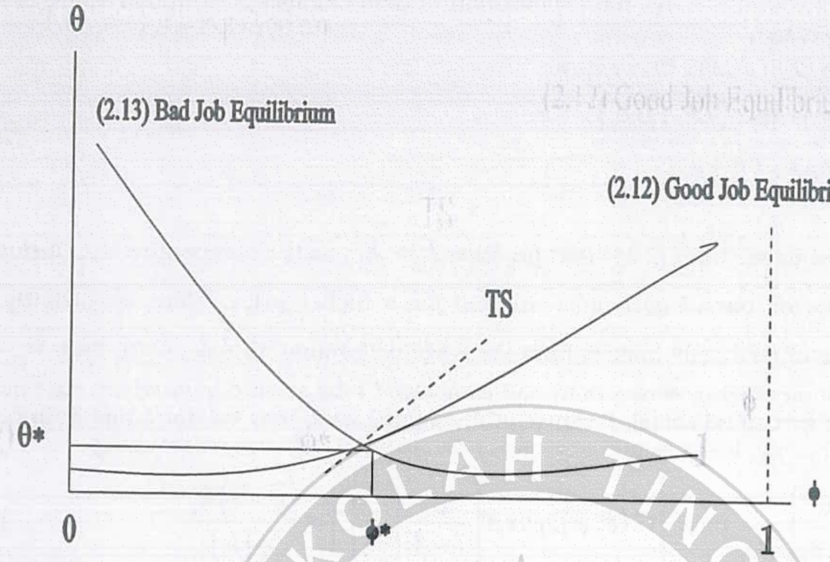

Figure 2.1. Job Equlibrium

Equation (2.13), the bad job equilibrium locus, cannot be shown to be decreasing everywhere. Intuitively, an increase in ϕ reduces p_b, thus requires a fall in θ to equilibrate the market. Therefore, (2.13) can be expected to be downward sloping. However, the general equilibrium effect through J^U (i.e. that a fall in ϕ reduces J^U) counteracts this and may dominate. Nevertheless, it is straight-forward to see that as ϕ tends to 1, (2.12) must be above (2.13) since at $\phi = 1$, (2.12) gives $\theta \to \infty$. Further, as ϕ goes to zero, (2.13) will give $\theta \to \infty$. Then by the continuity of the two functions they must intersect at least once in the range $\phi \in (0,1)$. Also since (2.13) can be upward sloping over some range, more than one inter sections are possible. Hence multiple equilibria cannot be ruled out.

3. Conclusions

The paper argues job composition is endogenously determined, and is highly responsive to labor market regulations. The key result is that in an unregulated equilibrium, there will be too small a proportion of good jobs and too many bad jobs. Minimum wages and unemployment benefits can improve this situation by encouraging workers to wait for better jobs, thus reducing the probability of bad jobs. This shifts the composition of employment towards good jobs.

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